

Answer the following questions:

1. Apply Descartes's rule of signs, find the nature of the roots of the equation:
$$x^4 + 2x^2 + 3x - 2 = 0$$
2. Solve by Cardan's method: $x^3 + 9x^2 + 15x - 25 = 0$
3. Find all the values of $(1 + i)^{\frac{2}{3}}$ and find the continued product of these values.
4. Solve the equation by Ferrari's method: $2x^4 + 6x^3 - 3x^2 + 2 = 0$
5. If α, β, ν, μ be the roots of the equation $x^4 - 3x^3 + 4x^2 - 5x + 6 = 0$, show that the value of $(\alpha^2 + 3)(\beta^2 + 3)(\nu^2 + 3)(\mu^2 + 3)$ is 57.
6. Define row reduced echelon form of a matrix. Reduce the following matrix to a row reduced echelon matrix and hence find the rank:
$$\begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$$
7. Show that $8xyz < (1-x)(1-y)(1-z) < \frac{8}{27}$, where $x+y+z=1$
8. Determine the conditions for which the system of equations:
$$x+y+z = 1$$
$$x+2y-z = b$$
$$5x+7y+az = b^2$$
Admits of (i) only one solution (ii) no solution (iii) many solutions.